WM-01-115
February 1, 2008

Extra neutral gauge bosons and Higgs bosons in an E_6-based model

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Extra neutral gauge bosons and Higgs bosons in an effective low-energy $SU(2)_L \times SU(2)_I \times U(1)_Y \times U(1)_{Y'}$ model, which is a subgroup of E_6, are studied. $SU(2)_I$ is a subgroup of $SU(3)_R$ and commutes with the electric charge operator, so the three corresponding gauge bosons are neutral. Electroweak precision experiments are used to put constraints on masses of the extra neutral gauge bosons and on the mixings between them and the ordinary Z boson, including constraints arising from a proposed measurment of the weak charge of the proton at Jefferson Lab. Bounds on and relationships of masses of Higgs bosons in the supersymmetric version of the model are also discussed.

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1 Introduction

The mass of the Higgs boson of the Standard Model(SM) is still undertemined, although there are recent reports indicating the observation of signals at LEP [1-3]. The requirement that the vacuum is stable and the perturbation is valid up to a large scale(for example, grand unification scale) can bound the mass(es) of Higgs boson(s) [4]. Extra Higgs bosons and gauge bosons will appear naturally in many extensions of the SM. Generally the masses of extra gauge bosons remain unpredicted and may or may not be of the order of the electroweak scale. The closeness of the observed W and Z boson properties with the predictions of the SM do not yield any direct information about the masses of extra gauge bosons, but seems to imply that the mixings of W or Z with extra gauge bosons should be very small.

E_6 models have been studied widely [5]. The maximal subgroup decomposition of E_6 containing QCD as an explicit factor is $SU(3)_C \times SU(3)_L \times SU(3)_R$, from which an effective low energy model $SU(2)_L \times SU(2)_I \times U(1)_Y \times U(1)_{Y'}$ can arise [6]. $SU(2)_I$ commutes with the electric charge operator and the corresponding gauge bosons are neutral. The most extensive works on the phenomenology of this model focused on the production of the W_I's in hadron-hadron, e^+e^-, and ep colliders [7, 8]. The t-channel production of exotic fermions in the model has recently been considered in Ref. [9]. In this paper we will study the gauge boson and Higgs boson sectors of the model, and bounds on the masses and mixings of extra neutral gauge bosons and Higgs bosons will be found.

In Ref. [10], a direct search for extra gauge bosons was reported and lower mass limits of approximately $500 \sim 700$ GeV were set, depending on the Z' couplings. The discovery potential and diagnostic abilities of proposed future colliders for new neutral or charged gauge bosons were summarized in Ref. [11]. Even though there is as yet no

direct experimental evidence of extra gauge bosons, stringent indirect constraints can be put on the mixings and the masses of extra gauge bosons by electroweak precision data. In Ref. [12-14], such constraints were derived in the $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ model. The lower mass limits were generally several hundreds of GeV and were competitive with experimental bounds from direct searches. A good summary of Z' searches can be found in Ref. [15] and references therein.

The paper is organized as follows. In the next section, the model will be described briefly, and a specific Higgs field assignment to break $SU(2)_L \times U(1)_Y \times SU(2)_I \times U(1)_{Y'}$ into $U(1)_{em}$ will be introduced. Sec. 3 deals with the extra neutral gauge bosons. The mixing among neutral gauge bosons will be discussed. In Sec. 4, electroweak precision experiments, including Z-pole experiments, m_W measurements and low-energy neutral current(LENC) experiments will be presented, with special attention being paid to a proposed measurement of the weak charge of the proton at Jefferson Lab. In Sec. 5, constraints on the masses of extra neutral gauge bosons and mixings will be found. In Sec. 6, bounds on and relationships of masses of Higgs bosons appearing in the supersymmetric version of the model will be derived. Sec. 7 contains our conclusions. Mass-squared matrices of neutral gauge bosons and Higgs bosons in the model are given in the Appendices.

2 The Model

There are many phenomenologically acceptable low energy models which can arise from E_6:

$$\begin{aligned}
(a) \quad E_6 &\rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\eta, \\
(b) \quad E_6 &\rightarrow SO(10) \times U(1)_\psi \rightarrow SU(5) \times U(1)_\chi \times U(1)_\psi, \\
(c) \quad E_6 &\rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R,
\end{aligned}$$

$$(c') \quad E_6 \quad \to SU(3)_C \times SU(2)_L \times SU(2)_I \times U(1)_Y \times U(1)_{Y'}, \tag{1}$$

where there is only one extra Z, generally called Z_η, in model (a). $U(1)_\psi$ and $U(1)_\chi$ can be combined into $U(1)_\theta$ as $Z'(\theta) = Z_\psi \cos\theta - Z_\chi \sin\theta$ in model (b), reducing it to the effective rank-5 model $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\theta$, which is most often considered in the literature. In particular, $U(1)_\eta$ corresponds to $\theta = \arcsin\sqrt{3/8}$. Model (c) and ($c'$) come from the subgroup $SU(3)_C \times SU(3)_L \times SU(3)_R$. The **27**-dimensional fundamental representation of E_6 has the branching rule

$$\mathbf{27} = \underbrace{(\mathbf{3}^c, \mathbf{3}, \mathbf{1})}_{q} + \underbrace{(\overline{\mathbf{3}}^c, \mathbf{1}, \overline{\mathbf{3}})}_{\overline{q}} + \underbrace{(\mathbf{1}^c, \overline{\mathbf{3}}, \mathbf{3})}_{l}\ , \tag{2}$$

and the particles of the first family are assigned as

$$\begin{pmatrix} u \\ d \\ h \end{pmatrix} + \begin{pmatrix} u^c & d^c & h^c \end{pmatrix} + \begin{pmatrix} E^c & \nu & N \\ N^c & e & E \\ e^c & \nu^c & S^c \end{pmatrix}, \tag{3}$$

where $SU(3)_L$ operates vertically and $SU(3)_R$ operates horizontally. (Different symbols for these particles may be used in the literature.)

The most common pattern of breaking the $SU(3)_R$ factor is to break the **3** of $SU(3)_R$ into **2**+**1**, so that (u^c, d^c) forms an $SU(2)_R$ doublet with h^c as a $SU(2)_R$ singlet. This gives model (c), the familiar left-right symmetric model [16]. Model (c) can be reduced further to an effective rank-5 model with $U(1)_{V=L+R}$. Another possibility, resulting in model (c'), is to break the **3** of the $SU(3)_R$ into $\mathbf{1} + \mathbf{2}$ so that (d^c, h^c) forms an $SU(2)$ doublet with u^c as a singlet. In this option, the $SU(2)$ doesn't contribute to the electromagnetic charge operator and it is called $SU(2)_I$ (I stands for Inert). Then the vector gauge bosons corresponding to $SU(2)_I$ are neutral.

At the $SU(2)_L \times SU(2)_I \times U(1)_Y \times U(1)_{Y'}$ level, a single generation of fermions can be represented as

$$\begin{pmatrix} \nu & N \\ e^- & E^- \end{pmatrix}_L, \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} d^c & h^c \end{pmatrix}_L, \begin{pmatrix} E^c \\ N^c \end{pmatrix}_L, \begin{pmatrix} \nu^c & S^c \end{pmatrix}_L, \ h_L, \ e^c_L, \ u^c_L, \tag{4}$$

Table **1** The quantum numbers of fermions in **27** of E_6

at the $SU(2)_L \times SU(2)_I \times U(1)_Y \times U(1)_{Y'}$ level.

State	T_{3L}	T_{3I}	Y	Y'	$Q_{em} = T_{3L} + Y/2$
u	1/2	0	1/3	2/3	2/3
d	-1/2	0	1/3	2/3	-1/3
u^c	0	0	-4/3	2/3	-2/3
d^c	0	1/2	2/3	-1/3	1/3
h	0	0	-2/3	-4/3	-1/3
h^c	0	-1/2	2/3	-1/3	1/3
e^-	-1/2	1/2	-1	-1/3	-1
e^c	0	0	2	2/3	1
E^-	-1/2	-1/2	-1	-1/3	-1
E^c	1/2	0	1	-4/3	1
ν	1/2	1/2	-1	-1/3	0
ν^c	0	1/2	0	5/3	0
N	1/2	-1/2	-1	-1/3	0
N^c	-1/2	0	1	-4/3	0
S^c	0	-1/2	0	5/3	0

where $SU(2)_{L(I)}$ acts vertically (horizontally). The quantum numbers of particles are listed in Table **1**.

In Ref. [17] the Higgs structure necessary to break $SU(2)_L \times SU(2)_I \times U(1)_Y \times U(1)_{Y'}$ down to $U(1)_{em}$ was discussed. The Higgs multiplets are

$$H_2 \equiv \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}, \quad \mathcal{H} \equiv \begin{pmatrix} H_1^0 & \tilde{\nu} \\ H_1^- & \tilde{e^-} \end{pmatrix}, \quad N \equiv \begin{pmatrix} N_2 & N_1 \end{pmatrix}, \quad N' \equiv \begin{pmatrix} N_2' & N_1' \end{pmatrix}, \tag{5}$$

with $SU(2)_L$ acting in the vertical direction and $SU(2)_I$ acting in the horizational direction. The U(1) quantum numbers are: $Y(H_2) = 1, Y(\mathcal{H}) = -1, Y(N) = Y(N') = 0$, and $Y'(H_2) = 4/3, Y'(\mathcal{H}) = 1/3, Y'(N) = Y'(N') = -5/3$. The doublets N and N' are also neutral. Note that two N doublets are needed. The reason can be seen in the limit where the model is broken down to the SM at a scale much greater than the electroweak scale. A single N doublet can only break $SU(2)_I \times U(1)_{Y'}$ down to U(1), leaving an extra unbroken U(1) symmetry.

The multiplets can get vacuum expectation values in the following way,

$$\langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \ \langle \mathcal{H} \rangle = \begin{pmatrix} v_1 & v_3 \\ 0 & 0 \end{pmatrix}, \ \langle N \rangle = \begin{pmatrix} n_2 & n_1 \end{pmatrix}, \ \langle N' \rangle = \begin{pmatrix} n'_2 & n'_1 \end{pmatrix}. \qquad (6)$$

Since we are not considering the spontaneous CP violation, the phase of the Higgs fields can be chosen such that all of vacuum expectation values are real and positive. There appear to be seven vacuum expectation values in the model, but one of them can be set to zero by performing an $SU(2)_I$ rotation. So there are only six physically relevant vacuum expectation values.

3 Extra neutral gauge bosons and mixings

In the $SU(2)_L \times U(1)_Y \times SU(2)_I \times U(1)_{Y'}$ model, the neutral gauge fields include the ordinary Z coming from $SU(2)_L \times U(1)_Y$; W_I^1, W_I^2 and W_I^3 for the $SU(2)_I$ group and B for $U(1)_{Y'}$. (We will use linear combinations $W_I^{\pm} = (W_I^1 \mp iW_I^2)/\sqrt{2}$ instead of W_I^1 and W_I^2, here $\pm$ is just a convention as they are neutral.) After the spontaneous symmetry breaking mechanism described in the previous section, the mass-squared matrix for the neutral gauge bosons is a symmetric 5×5 matrix, whose elements are listed in the appendices.

It is apparent that there are mixings among the neutral gauge bosons. It is impossible to diagonalize the matrix analytically. Numerical calculations must be needed to get the eigenstates and corresponding eigenvalues.

It is noted that the elements in the first row(column) are independent of the vacuum expectation values n_i and n'_i(i=1,2). Therefore when they are very large, the mixing should be small. In this decoupling limit, the only observable neutral gauge boson is the ordinary Z and its mass should be the exact value measured experimentally. The extra neutral gauge bosons are not yet accessible experimentally, but their existence will have effects in electroweak radiative corrections.

In order to find mass eigenstates and mixing angles, the mass-squared matrix $\mathcal{M}^2$ can be split into two parts

$$\begin{aligned}
\mathcal{M}^2 &= \mathcal{M}_1^2 + \mathcal{M}_2^2 \\
&= \begin{pmatrix} m_Z^2 & 0 & 0 & 0 & 0 \\ 0 & m_{W_I^3}^2 & m_{23} & m_{24} & m_{25} \\ 0 & m_{23} & m_B^2 & m_{34} & m_{35} \\ 0 & m_{24} & m_{34} & 0 & m_{W_I^\pm}^2 \\ 0 & m_{25} & m_{35} & m_{W_I^\pm}^2 & 0 \end{pmatrix} + \begin{pmatrix} 0 & m_{12} & m_{13} & m_{14} & m_{15} \\ m_{12} & 0 & 0 & 0 & 0 \\ m_{13} & 0 & 0 & 0 & 0 \\ m_{14} & 0 & 0 & 0 & 0 \\ m_{15} & 0 & 0 & 0 & 0 \end{pmatrix} .
\end{aligned} \tag{7}$$

First we can use a 5×5 unitary matrix $\mathbf{U}_1$ to diagonalize $\mathcal{M}_1^2$, and $\mathbf{U}_1$ can have the form

$$\mathbf{U}_1 = \begin{pmatrix} 1 & 0 \\ 0 & \mathbf{u}_1 \end{pmatrix} , \tag{8}$$

where $\mathbf{u}_1$ is a 4×4 unitary matrix. This is to find mass eigenstates of extra neutral gauge bosons. There is no mixing of ordinary Z boson with extra neutral gauge bosons at this stage. Then the total mass-squared matrix for the neutral gauge bosons under the new basis has the form

$$\begin{aligned}
\mathcal{M}'^2 &= \mathcal{M}_1'^2 + \mathcal{M}_2'^2 \\
&= \begin{pmatrix} m_Z^2 & m'_{12} & m'_{13} & m'_{14} & m'_{15} \\ m'_{12} & m_{Z_2}^2 & & & \\ m'_{13} & & m_{Z_3}^2 & & \\ m'_{14} & & & m_{Z_4}^2 & \\ m'_{15} & & & & m_{Z_5}^2 \end{pmatrix} .
\end{aligned} \tag{9}$$

$\mathcal{M}'^2$ can be principally diagonalized by another unitary matrix $\mathbf{U}_2$, then we can get a unitary matrix $\mathbf{U} = \mathbf{U}_2 \times \mathbf{U}_1$ which can be used to diagonalize the original matrix $\mathcal{M}^2$. The mixings of ordinary Z boson with extra neutral gauge bosons occur in this transformation. For small mixings, the elements of $\mathbf{U}_2$ will have the following properties

$$\begin{aligned}
(\mathbf{U}_2)_{11} &\sim 1.0, \\
(\mathbf{U}_2)_{j1} &\sim \left(\frac{m_Z^2 - m_{Z_1}^2}{m_{Z'_j}^2 - m_Z^2}\right)^{1/2}, \\
(\mathbf{U}_2)_{jk} &\sim 0, \qquad j \neq k.
\end{aligned} \tag{10}$$

Therefore $(\mathbf{U}_2)_{j1}$ can be treated as effective mixing angles.

The couplings between neutral gauge bosons and fermions, which will give neutral current processes, are

$$\begin{aligned}\mathcal{L}_{NC} &= -\sum_{f,\alpha}\{g_Z\overline{f}_\alpha\gamma^\mu\left(T_{3L}^{f_\alpha} - Q_{f_\alpha}\sin^2\theta_W\right)f_\alpha Z_\mu + g_{Y'}Y'_{f_\alpha}/2\overline{f}_\alpha\gamma^\mu f_\alpha B_\mu \\ &\quad + g_I T_{3I}^{f_\alpha}\overline{f}_\alpha\gamma^\mu f_\alpha W_{I\mu}^3\},\end{aligned} \tag{11}$$

where the first term in the brackets represents the SM neutral currents, the second and third terms represent additional neutral currents introduced by extra neutral gauge bosons, and $g_Z = g_L/\cos\theta_W = g_Y/\sin\theta_W$. The symbol f_α denotes the leptons or quarks with the chirality α ($\alpha = L$ or R). The quantum numbers $T_{3L}^{f_\alpha}$, Q_{f_α}, Y'_{f_α} and $T_{3I}^{f_\alpha}$ can be read from Table **1**. The flavor-changing neutral currents caused by $W_I^\pm$ involve heavy fermions and will not be included here.

After the $\mathbf{U}_1$-transformation, the interaction Lagrangian changes as

$$\begin{aligned}\mathcal{L}_{NC} &= -\sum_{f,\alpha}\{g_Z\overline{f}_\alpha\gamma^\mu\left(T_{3L}^{f_\alpha} - Q_{f_\alpha}\sin^2\theta_W\right)f_\alpha Z_\mu \\ &\quad + g_{Y'}Y'_{f_\alpha}/2\overline{f}_\alpha\gamma^\mu f_\alpha\sum_{j\neq 1}(\mathbf{U}_1)_{3j}Z_{j\mu} \\ &\quad + g_I T_{3I}^{f_\alpha}\overline{f}_\alpha\gamma^\mu f_\alpha\sum_{j\neq 1}(\mathbf{U}_1)_{2j}Z_{j\mu}\}.\end{aligned} \tag{12}$$

where the first term is unchanged because there is no mixing of ordinary Z boson with extra neutral gauge bosons. Considering the $\mathbf{U}_2$-transformation, the final interaction Lagrangian is given as

$$\begin{aligned}\mathcal{L}_{NC} &= -\sum_{f,\alpha}\{g_Z\overline{f}_\alpha\gamma^\mu\left(T_{3L}^{f_\alpha} - Q_{f_\alpha}\sin^2\theta_W\right)f_\alpha[(\mathbf{U}_2)_{11}Z_{1\mu} + \sum_{j\neq 1}(\mathbf{U}_2)_{1j}Z'_{j\mu}] \\ &\quad + g_{Y'}Y'_{f_\alpha}/2\overline{f}_\alpha\gamma^\mu f_\alpha\sum_{j\neq 1}(\mathbf{U}_1)_{3j}[(\mathbf{U}_2)_{j1}Z_{1\mu} + \sum_{k\neq 1}(\mathbf{U}_2)_{jk}Z'_{k\mu}] \\ &\quad + g_I T_{3I}^{f_\alpha}\overline{f}_\alpha\gamma^\mu f_\alpha\sum_{j\neq 1}(\mathbf{U}_1)_{2j}[(\mathbf{U}_2)_{j1}Z_{1\mu} + \sum_{k\neq 1}(\mathbf{U}_2)_{jk}Z'_{k\mu}]\},\end{aligned} \tag{13}$$

The contributions from the term $(\mathbf{U}_2)_{1j}Z'_{j\mu}$ can be omitted in our analysis because they

are combinations of mixings and exchanges of extra neutral gauge bosons and should be very small.

Due to the mixings, the mass, m_{Z_1} of the observed Z boson is shifted from the SM prediction m_Z.

$$\Delta m^2 \equiv m_{Z_1}^2 - m_Z^2 \leq 0. \tag{14}$$

The presence of this mass shift will affect the T-parameter [18] at tree level. From Ref. [14], the T-parameter is expressed in terms of the effective form factors $\overline{g}_Z^2(0)$, $\overline{g}_W^2(0)$ and the fine structure constant α as

$$\begin{aligned} \alpha T &\equiv 1 - \frac{\overline{g}_W^2(0)}{m_W^2} \frac{m_{Z_1}^2}{\overline{g}_Z^2(0)} \\ &= \alpha(T_{SM} + T_{new}), \end{aligned} \tag{15}$$

where T_{SM} and the new physics contribution T_{new} are given by

$$\alpha T_{SM} = 1 - \frac{\overline{g}_W^2(0)}{m_W^2} \frac{m_Z^2}{\overline{g}_Z^2(0)}, \tag{16}$$

$$\alpha T_{new} = -\frac{\Delta m^2}{m_{Z_1}^2} \geq 0. \tag{17}$$

It is noted that the positiveness of T_{new} is attributed to the mixings which always lower the mass of the ordinary Z boson. The effects of Z-Z' mixings can be described by the effective mixing angles and the positive T_{new}.

4 Electroweak observables

The experimental data used to put indirect constraints on extra neutral gauge bosons are summarized in Table **2**. The data includes the Z-pole experiments, the W boson mass measurement and LENC experiments. They are updated from Ref. [19, 20]. The family universality is assumed in our analysis.

Table **2** Summary of precision electroweak measurements used in our analysis.

Z-pole experiments	
m_Z (GeV)	91.1872 ± 0.0021
Γ_Z (GeV)	2.4944 ± 0.0024
σ_h^0 (nb)	41.544 ± 0.037
R_l	20.784 ± 0.023
$A_{FB}^{0,l}$	0.0170 ± 0.0009
A_τ	0.1425 ± 0.0043
A_e	0.1511 ± 0.0019
R_b	0.21642 ± 0.00073
R_c	0.1674 ± 0.0038
$A_{FB}^{0,b}$	0.0988 ± 0.0020
$A_{FB}^{0,c}$	0.0692 ± 0.0037
A_{LR}^0	0.1495 ± 0.0017
A_b	0.911 ± 0.025
A_c	0.630 ± 0.026
W-mass measurement	
m_W (GeV)	80.394 ± 0.042
LENC experiments	
A_{SLAC}	0.80 ± 0.058
A_{CERN}	-1.57 ± 0.38
A_{Bates}	-0.137 ± 0.033
A_{Mainz}	-0.94 ± 0.19
$Q_W(^{133}_{55}\mathrm{Cs})$	-72.06 ± 0.44
K_{FH}	0.3247 ± 0.0040
K_{CCFR}	0.5820 ± 0.0049
$g_{LL}^{\nu_\mu e}$	-0.269 ± 0.011
$g_{LR}^{\nu_\mu e}$	0.234 ± 0.011

In addition to the electroweak observables generally used in the literature, we also consider the possible constraint arising from the weak charge of the proton, which is proposed to be measured at Jefferson Lab. In contrast to the weak charge of a heavy atom, the weak charge of the proton is fortuitously suppressed in the SM. Therefore it is very sensitive to the contributions from new physics. Additionally it is twice as sensitive to new u-quark interactions as it is to new d-quark physics. In the model considered here the right-handed u-quark and d-quark have different isospin contents under $SU(2)_I$, so

it is advantageous to consider the constraints arising from the anticipated measurement. The theoretical prediction [14] for the weak charge of the proton can be derived

$$Q_W^P = 0.07202 - 0.01362\Delta S + 0.00954\Delta T + 2(2\Delta C_{1u} + \Delta C_{1d}). \tag{18}$$

5 Constraints on extra neutral gauge bosons

Using the electroweak precision data, constraints on mixing angles and masses of extra neutral gauge bosons can be obtained from the standard χ^2 analysis. For simplicity, S_{new} and U_{new} will be set zero because they are very small. Through our analysis, we will use precisely determined parameters m_{Z_1}, G_f and $\overline{\alpha}(m_{Z_1}^2)$ as inputs. The Higgs mass dependence of the results are ignored for simplicity. We set the top quark mass $m_t = 175$ GeV and Higgs boson mass $m_H = 100$ GeV in our analysis. We first obtain the constraints from Z-pole experiments and m_W measurement only, and then we combine the LENC experiments with them to get further constraints. Finally we will study the possible constraints which would arise from measuring the weak charge of the proton.

5.1 Constraints from Z-pole and m_W data

From the previous analysis, it is found that the Z-pole experiments are related to mixings and the T-parameter, while m_W is only relevant for the T-parameter. If we set all mixing angles and T_{new} equal to zero, it will give the fit for the SM. It serves as a reference because the SM fits the experiments very well. Defining $\Delta\chi^2 = \chi^2 - \chi^2_{SM}$, by requiring acceptable $\Delta\chi^2$ we can get constraints on the mixings and the masses of extra neutral gauge bosons. The result for $\Delta\chi^2 = 1.0$ is illustrated in Fig. 1. The lower mass limit for the lightest extra gauge boson is about 400 GeV. It seems that the model allows for the existence of a comparatively light extra neutral gauge boson. But we will find in the following that this is not true when LENC experiments are included. The mixing angles are found to be very small, namely $|\theta| \leq 0.003$.

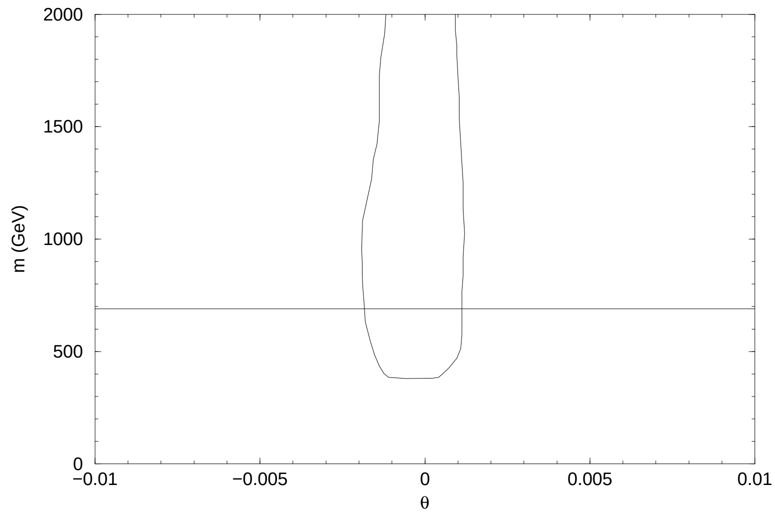

Figure 1: The contour of $\Delta\chi^2 = \chi^2 - \chi^2_{SM} = 1.0$ for the lightest extra neutral gauge boson. The constraint is obtained by use of Z-pole experiments and m_W measurement. As a reference the lower direct production limit from CDF [10] for the sequential Z_{SM} is also shown.

The sequential Z_{SM} boson [21] is defined to have the same couplings to fermions as the SM Z boson. Such a boson is not expected in the context of gauge theories unless it has different couplings to exotic fermions than the ordinary Z. However, it serves as a useful reference case when comparing constraints from various sources. The direct production limit for the sequential Z_{SM} boson from Ref. [10] is about 690 GeV. It is assumed that all exotic decay channels are forbidden, and the bound has to be relaxed by about 100 to 150 GeV when all exotic decays(including channels involving superparticles) are kinetically allowed. It is found that, at this time, the lower mass limit for the lightest extra neutral gauge boson is much lower than the direct production limit for the sequential Z_{SM} boson.

5.2 Constrains from Z-pole + m_W + LENC data

The LENC experiments can get contributions from the exchanges of extra neutral gauge bosons, which can be approximated by contact interactions. The contact interactions are inversely-proportional to the masses of the extra gauge bosons exchanged in

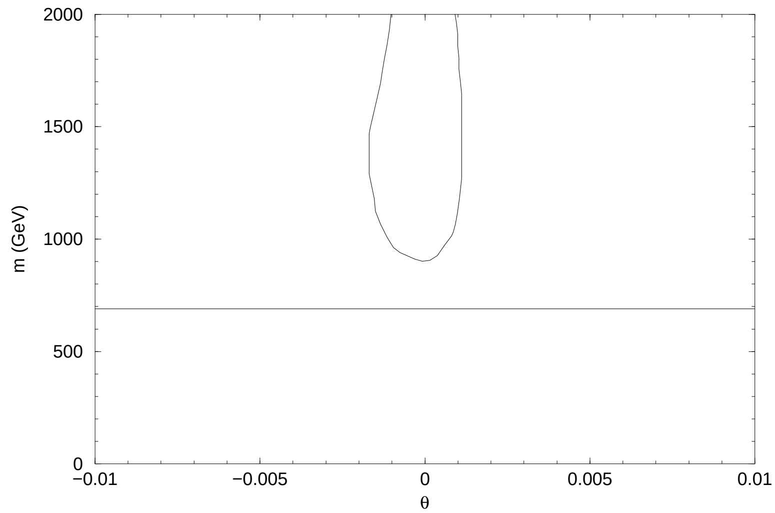

Figure 2: The contour of $\Delta\chi^2 = \chi^2 - \chi^2_{SM} = 1.0$ for the lightest extra neutral gauge boson. The constraint is obtained by use of Z-pole experiments, m_W measurement and LENC experiments. As a reference the lower direct production limit from CDF [10] for the sequential Z_{SM} is also shown.

the processes. So the LENC experiments can put stringent constraints on the masses of extra neutral gauge bosons. The results of fitting Z-pole experiments, m_W measurement and LENC experiments are shown in Fig. 2. The lower mass limits for the extra neutral gauge bosons are raised much higher than those without LENC experiments. The lower mass bound for the lightest extra gauge boson is about 900 GeV. It is higher than the direct production limit for the sequential Z_{SM} boson.

In Ref. [13], similar constraints on various possible extra Z' bosons were studied. In all cases the mixing angles are severely constrained($\sin\theta < 0.01$), and the lower mass limit are generally of the order of several hundred GeV, depending on the specific models considered.

In the model considered here, from the appendices, $m^2_{W^3_I} \sim m^2_B$ assuming that $g_I = g_L$ and $g_{Y'} = g_Y$. It is apparent that $m_{W^\pm_I}$ is degenerate with $m_{W^3_I}$ without mixing. Generally the lightest extra neutral gauge boson mainly consists of W^3_I, or Z_I. It is noted that Z_I corresponds to $Z'(\theta = -\arcsin\sqrt{5/8})$ and is orthogonal to Z_η. There is

no mass limit on Z_I from electroweak precision data available in the literature. From constraints on Z_ψ, Z_χ and Z_η [13], it could be inferred that the mass limit on Z_I would be about 430 GeV at 95% CL. In Ref. [10] the lower mass limit of 565 GeV for Z_I was set by direct search for heavy neutral gauge bosons with the Collider Detector at Fermilab. Our mass limit on the lightest extra neutral gauge boson is much higher mainly due to more updateded data used in our analysis.

It should be pointed out that an updated value for $Q_W(\mathrm{Cs}) = -72.06(28)_{\mathrm{expt}}(34)_{\mathrm{theor}}$ has been reported [22]. The experimental precision was improved and indicated a 2.5σ deviation from the prediction of the SM. The possibility that the discrepancy is due to contributions from new physics has been suggested. In Ref. [23, 24] it was shown that the contribution from the exchange of an extra U(1) boson could explain the data without $Z - Z'$ mixing. Some models which would give negative contibutions to $Q_W(\mathrm{Cs})$, such as Z_{SM} and Z_η, were excluded at 99% CL. The existance of Z_I with a central value of about 760 GeV could explain the deviation.

Of cousre, a 2.5σ discrepance is insufficient to claim a discovery, so we have used the data to determine lower mass bounds and mixings of additional neutral gauge bosons. It put much stronger constraints on the mass and mixing of the lightest extra neutral gauge boson than the old data.

From Ref. [11] the typical bounds achievable on extra neutral or charged gauge bosons $m_{Z'(W')}$ at the coming colliders such as Tevatron, LHC, 500 GeV NLC and 1 TeV NLC are approximately 1 TeV, 4 TeV, 1-3 TeV and 2-6 TeV correspondingly. Therefore the extra neutral gauge bosons in the model could be studied well in the coming colliding experiments.

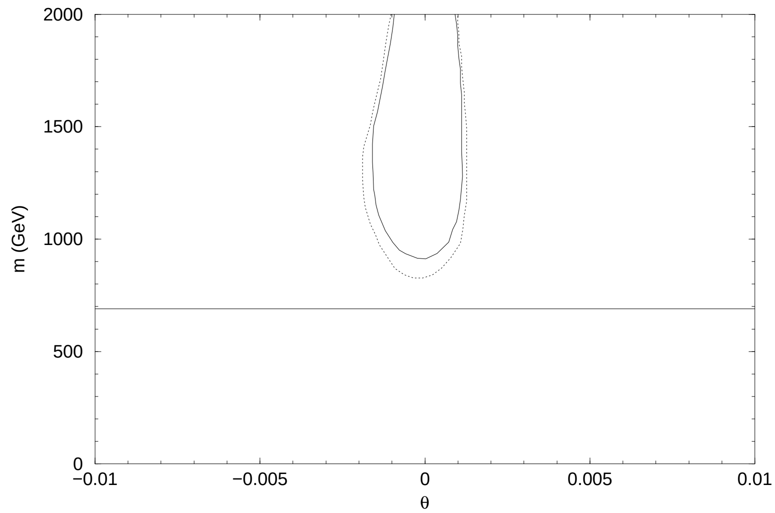

Figure 3: The contour of $\Delta\chi^2 = \chi^2 - \chi^2_{SM} = 1.0$ for the lightest extra neutral gauge boson with the data of Z-pole experiments, m_W measurement, LENC experiments and proposed measurement of the weak charge of the proton with the precision level of 3%. As a reference the lower direct production limit from CDF [10] for the sequential Z_{SM} is also shown. The contour of $\Delta\chi^2 = 2.0$ is also shown(dotted line).

5.3 Constrains from Z-pole + m_W + LENC data + Q_W^P

In Ref. [25], it is proposed to measure the weak charge of the proton, Q_W^P, with parity-violating ep scattering at $Q^2 = 0.03(\mathrm{GeV/c})^2$ at Jefferson Lab. A high statistical accuracy is expected to be achieved with the current facility. Specifically, $\Delta Q_W^P / Q_W^{0P} \sim 4\%$ or better is possible. Fig. 3 illustrates the constraints on the lightest extra neutral gauge boson including the Q_W^P assuming that the precision level is 3%. It is found that the lower mass bound of the the lightest extra neutral gauge boson is almost same as the constraint with the data of Z-pole expreiments, m_W and LENC experiments. Should the weak charge of the proton be measured with a high precision level, it would yield competitive constraints on the model.

In Ref. [26], the new physics sensitivity of a variety of low energy parity-violating observables was analyzed. Taken as an example, present and prospective mass limits on an additional gauge boson, Z_χ, were given. Were the precison of measuring the

weak charge of the proton 10%(3%), the lower bound would be 585(1100) GeV. This is compatible with our result.

6 Bounds on Higgs bosons

There are a large number of Higgs bosons in the model: 6 scalar, 3 pseudoscalar and 4 charged Higgs bosons. In general, the scalar potential will have too many parameters to make any meaningful statement about masses of Higgs bosons. However, in the supersymmetric version of the model, the scalar potential is highly constrained.

The most general superpotential satisfying gauge invariance can be written as

$$W = \lambda H_2 \mathcal{H} N + \lambda' H_2 \mathcal{H} N'. \tag{19}$$

Here $H_2\mathcal{H}N$ means $\varepsilon_{ij} {H_2}^i \mathcal{H}^{\alpha j} \varepsilon_{\alpha\beta} N^\beta$, i, j are $SU(2)_L$ indices and α, β are $SU(2)_I$ indices. The scalar potential is given by

$$V = V_F + V_D + V_{soft}, \tag{20}$$

where

$$V_F = \sum_i |\, \partial W / \partial \phi_i|^2 \tag{21}$$

is the F-term, the sum runs over all complex scalar ϕ_i's appearing in the theory.

$$V_D = 1/2 \sum_a |\sum_i (g_a \phi_i^\dagger T^a \phi_i) + \xi_a|^2 \tag{22}$$

is the D-term, T^a represent generators of corresponding gauge groups and g_a coupling constants. The ξ terms only exist if a labels a U(1) generator, and in our consideration they are set to zero for simplicity.

$$\begin{aligned} V_{soft} \;=\;& m_{\mathcal{H}}^2 Tr(\mathcal{H}^\dagger \mathcal{H}) + m_{H_2}^2 H_2^\dagger H_2 + m_N^2 N^\dagger N + m_{N'}^2 N'^\dagger N' \\ & -\lambda A (H_2 \mathcal{H} N + h.c.) - \lambda' A' (H_2 \mathcal{H} N' + h.c.) - m_3^2 (N^\dagger N' + h.c.) \end{aligned} \tag{23}$$

are soft supersymmetry breaking terms. The soft supersymmetry breaking parameters will be considered completely arbitrary, therefore we only study the tree-level potential. The radiative corrections to the potential will not significantly affect the results because the primary effects of the radiative corrections are to change the effective soft supersymmetry breaking terms. The exception is due to top quark contribution, proportional to m_{top}^4, and it will increase some mass limits by up to 20 GeV.

The complete potential has nine parameters: λ, λ', the coefficients of the two trilinear terms A and A', the four mass-squared parameters $m_{\mathcal{H}}^2$, $m_{H_2}^2$, m_N^2 and $m_{N'}^2$, and m_3^2. Six of them can be transferred to vacuum expectation values, thus three undetermined parameters remain, which we take to be λ, λ' and m_3^2. All the parameters are chosen to be real, therefore the scalar potential is CP invariant.

It is straightforward but tedious to work out the mass-squared matrices for various Higgs bosons, which are given in the appendices. The mass-squared matrices for the neutral scalars and pseudoscalars are 7×7 matrices. The two matrices are decoupled from each other because the scalar potential is CP invariant. The former must have one zero eigenvalue and the latter must have four zero eigenvalues, corresponding to the five Goldstone bosons eaten by the five massive neutral vector gauge bosons[the zero eigenvalue of the scalar mass-squared matrix corresponds to the freedom to perform an $SU(2)_I$ rotation in order to set one of neutral vacuum expectation values to zero]. The mass-squared matrices for charged Higgs scalars are 3×3 matrices. The positive states and negative states decouple, and they share the same mass-squared matrix. There is one zero eigenvalue for each of them in order to produce masses for two charged vector bosons of $SU(2)_L$. As we must resort to numerical techniques to find the eigenvalues of the Higgs bosons, the presence of the required number of zero eigenvalues provides an excellent check on our numerical calculation. As another check, we found that there

exists a relationship

$$TrM_{\phi}{}^2 = TrM_Z{}^2 + TrM_{H_3^0}{}^2, \tag{24}$$

where M_Z^2 is the neutral-vector mass-squared matrix, M_{ϕ}^2 is the neutral-scalar mass-squared matrix, and $M_{H_3^0}^2$ represents the pseudoscalar mass-squared matrix. This is a very general relation. It holds in any supersymmetric model based on an extended gauge group in which there are no gauge-singlet fields. Interestingly, in this model, the trace of the neutral-vector mass-squared matrix must include the W_I fields, which are the neutral nondiagonal bosons of the $SU(2)_I$ group.

For every set of values of λ, λ' and m_3^2, we searched numerically for the minimum of the scalar potential. We choose λ and λ' to be as large as 1 and m_3 to be as large as 1000 GeV. If the value of λ or λ' is too large, it will blow up at the unification scale by the renormalization group analysis as in the SM. Adjusting the various vacuum expectation values until the eigenvalues of the Higgs-boson mass matrices are positive or zero, we read off the value of the smallest nonzero eigenvalue of the neutral scalar mass-squared matrix. Then we vary the values of λ, λ' and m_3^2 to find the largest possible value of this smallest nonzero eigenvalue. We find that its value is about 150 GeV.

7 Conclusions

We have considered the effective low-energy $SU(2)_L \times U(1)_Y \times SU(2)_I \times U(1)_{Y'}$ model, which can arise from the E_6 unification model. The $SU(2)_I$ is a subgroup of $SU(3)_R$ and commutes with the electric charge operator, so the three corresponding gauge bosons are neutral. The gauge boson and Higgs boson sectors of the model are studied.

The extra neutral gauge bosons generally mix with each other and also with the ordinary Z boson. The electroweak precision data including Z-pole experiments, m_W measurement and LENC experiments are used to put constraints on masses of extra

gauge bosons and the mixings with ordinary Z bosons. The possible constraint from the weak charge of the proton, which is proposed to be measured at Jefferson Lab, is also considered. It is found that the mixings are very small, namely $|\theta| \leq 0.003$. The lower mass limit for the lightest extra neutral gauge boson is found to be about 900 GeV, which is somewhat higher than bounds in the literature mainly due to more updated data used in our analysis.

The scalar potential is highly constrained in the supersymmetric version of the model. An upper bound of about 150 GeV to the mass of the lightest CP-even Higgs scalar is found.

We thank John M. Finn for informing us of the proposal of the measurement of the weak charge of the proton at Jefferson Lab. This work was supported by the National Science Foundation grant NSF-PHY-9900657.

Appendix

A Mass-squared matrix for neutral gauge bosons

The mass-squared matrix for neutral gauge bosons is a symmetric 5×5 matrix.

$$
\begin{aligned}
m_Z^2 &= \frac{1}{4}(g_L^2 + g_Y^2)(v_1^2 + v_2^2 + v_3^2), \\
m_{12} &= \frac{1}{4}\sqrt{g_L^2 + g_Y^2}\, g_I (v_1^2 - v_3^2), \\
m_{13} &= \frac{1}{12}\sqrt{g_L^2 + g_Y^2}\, g_{Y'} (-v_1^2 + 4v_2^2 - v_3^2), \\
m_{14} &= m_{15} = \frac{1}{4}\sqrt{g_L^2 + g_Y^2}\, g_I v_1 v_3, \\
m_{W_I^3}^2 &= \frac{1}{4} g_I^2 (v_1^2 + v_3^2 + n_1^2 + n_2^2 + {n'_1}^2 + {n'_2}^2), \\
m_{23} &= \frac{1}{12} g_I g_{Y'} [-v_1^2 + v_3^2 - 5(n_1^2 - n_2^2 + {n'_1}^2 - {n'_2}^2)], \\
m_{24} &= m_{25} = 0, \\
m_B^2 &= \frac{1}{36} g_{Y'}^2 [v_1^2 + 16v_2^2 + v_3^2 + 25(n_1^2 + n_2^2 + {n'_1}^2 + {n'_2}^2)],
\end{aligned}
$$

$$
\begin{aligned}
m_{34} &= m_{35} = \frac{1}{12\sqrt{2}} g_I g_{Y'} [-v_1 v_3 + 5(n_1 n_2 + n'_1 n'_2)], \\
m_{44} &= m_{55} = 0, \\
m^2_{W_I^\pm} &= \frac{1}{4} g_I^2 (v_1^2 + v_3^2 + n_1^2 + n_2^2 + {n'_1}^2 + {n'_2}^2).
\end{aligned} \tag{25}
$$

B Various Higgs boson mass-squared matrices

The Higgs boson mass-squared matrix is obtained from

$$
M^2_{ij} = \frac{\partial^2 V}{\partial \phi_i \phi_j}|_{minimum} \ . \tag{26}
$$

B.1 Scalar Higgs boson mass-squared matrix

The mass-squared matrix for scalar Higgs bosons is a 7×7 symmetric matrix, S. Let

$$
V^2 \equiv v_1^2 + 4 v_2^2 + v_3^2 - 5(n_1^2 + n_2^2 + {n'_1}^2 + {n'_2}^2), \tag{27}
$$

$$
\begin{aligned}
S_{11} &= (\lambda n_1 + \lambda' n'_1)^2 + (\lambda^2 + \lambda'^2) v_2^2 + \frac{1}{2}(g_L^2 + g_Y^2 + g_I^2 + \frac{1}{9} g_{Y'}^2) v_1^2 + \frac{1}{4} g_L^2 (v_1^2 - v_2^2 + v_3^2) \\
&\quad + \frac{1}{4} g_Y^2 (v_1^2 - v_2^2 + v_3^2) + \frac{1}{4} g_I^2 (v_1^2 + v_3^2 + n_2^2 - n_1^2 + {n'_2}^2 - {n'_1}^2) + \frac{1}{36} g_{Y'}^2 V^2 + m^2_{\mathcal{H}}, \\
S_{12} &= 2(\lambda^2 + \lambda'^2) v_1 v_2 - \frac{1}{2}(g_L^2 + g_Y^2 - \frac{4}{9} g_{Y'}^2) v_1 v_2 + \lambda A n_1 + \lambda' A' n'_1, \\
S_{13} &= -(\lambda n_1 + \lambda' n'_1)(\lambda n_2 + \lambda' n'_2) + \frac{1}{2}(g_L^2 + g_Y^2 + g_I^2 + \frac{1}{9} g_{Y'}^2) v_1 v_3 + \frac{1}{2} g_I^2 (n_1 n_2 + n'_1 n'_2), \\
S_{14} &= -\lambda v_3 (\lambda n_1 + \lambda' n'_1) + \frac{1}{2} g_I^2 (v_1 n_2 + v_3 n_1) - \frac{5}{18} g_{Y'}^2 v_1 n_2, \\
S_{15} &= 2\lambda v_1 (\lambda n_1 + \lambda' n'_1) - \lambda v_3 (\lambda n_2 + \lambda' n'_2) + \frac{1}{2} g_I^2 (v_3 n_2 - v_1 n_1) - \frac{5}{18} g_{Y'}^2 v_1 n_1 + \lambda A v_2, \\
S_{16} &= -\lambda' v_3 (\lambda n_1 + \lambda' n'_1) + \frac{1}{2} g_I^2 (v_1 n'_2 + v_3 n'_1) - \frac{5}{18} g_{Y'}^2 v_1 n'_2, \\
S_{17} &= \lambda' v_1 (\lambda n_1 + \lambda' n'_1) + \lambda' [(\lambda n_1 + \lambda' n'_1) v_1 - (\lambda n_2 + \lambda' n'_2) v_3], \\
&\quad + \frac{1}{2} g_I^2 (v_3 n'_2 - v_1 n'_1) - \frac{5}{18} g_{Y'}^2 v_1 n'_1 + \lambda' A' v_2, \\
S_{22} &= (\lambda n_1 + \lambda' n'_1)^2 + (\lambda n_2 + \lambda' n'_2)^2 + (\lambda^2 + \lambda'^2)(v_1^2 + v_3^2) + \frac{1}{2}(g_L^2 + g_Y^2 + \frac{16}{9} g_{Y'}^2) v_2^2 \\
&\quad - \frac{1}{4} g_L^2 (v_1^2 - v_2^2 + v_3^2) + \frac{1}{4} g_Y^2 (v_1^2 - v_2^2 + v_3^2) + \frac{1}{9} g_{Y'}^2 V^2 + m^2_{H_2},
\end{aligned}
$$

$$
\begin{aligned}
S_{23} &= 2(\lambda^2+\lambda'^2)v_2v_3-\frac{1}{2}(g_L^2+g_Y^2-\frac{4}{9}g_{Y'}^2)v_2v_3-\lambda A n_2-\lambda' A' n_2',\\
S_{24} &= 2\lambda v_2(\lambda n_2+\lambda' n_2')-\frac{10}{9}g_{Y'}^2v_2n_2-\lambda A v_3,\\
S_{25} &= 2\lambda v_2(\lambda n_1+\lambda' n_1')-\frac{10}{9}g_{Y'}^2v_2n_1+\lambda A v_1,\\
S_{26} &= 2\lambda' v_2(\lambda n_2+\lambda' n_2')-\frac{10}{9}g_{Y'}^2v_2n_2'-\lambda' A' v_3,\\
S_{27} &= 2\lambda' v_2(\lambda n_1+\lambda' n_1')-\frac{10}{9}g_{Y'}^2v_2n_1'+\lambda' A' v_1,\\
S_{33} &= (\lambda n_2+\lambda' n_2')^2+(\lambda^2+\lambda'^2)v_2^2+\frac{1}{2}(g_L^2+g_Y^2+g_I^2+\frac{1}{9}g_{Y'}^2)v_3^2+\frac{1}{4}g_L^2(v_1^2-v_2^2+v_3^2)\\
&\quad +\frac{1}{4}g_Y^2(v_1^2-v_2^2+v_3^2)+\frac{1}{4}g_I^2(v_1^2+v_3^2+n_1^2-n_2^2+{n_1'}^2-{n_2'}^2)+\frac{1}{36}g_{Y'}^2V^2+m_{\mathcal{H}}^2,\\
S_{34} &= 2(\lambda n_2+\lambda' n_2')\lambda v_3-(\lambda n_1+\lambda' n_1')\lambda v_1+\frac{1}{2}g_I^2(v_1n_1-v_3n_2)-\frac{5}{18}g_{Y'}^2v_3n_2-\lambda A v_2,\\
S_{35} &= -(\lambda n_2+\lambda' n_2')\lambda v_1+\frac{1}{2}g_I^2(v_1n_2+v_3n_1)-\frac{5}{18}g_{Y'}^2v_3n_1,\\
S_{36} &= 2(\lambda n_2+\lambda' n_2')\lambda' v_3-(\lambda n_1+\lambda' n_1')\lambda' v_1+\frac{1}{2}g_I^2(v_1n_1'-v_3n_2')-\frac{5}{18}g_{Y'}^2v_3n_2'-\lambda' A' v_2,\\
S_{37} &= -(\lambda n_2+\lambda' n_2')\lambda' v_1+\frac{1}{2}g_I^2(v_1n_2'+v_3n_1')-\frac{5}{18}g_{Y'}^2v_3n_1',\\
S_{44} &= \lambda^2(v_2^2+v_3^2)+\frac{1}{2}(g_I^2+\frac{25}{9}g_{Y'}^2)n_2^2+\frac{1}{4}g_I^2(v_1^2-v_3^2+n_1^2+n_2^2-{n_1'}^2+{n_2'}^2)\\
&\quad -\frac{5}{36}g_{Y'}^2V^2+m_N^2,\\
S_{45} &= -\lambda^2v_1v_3+\frac{1}{2}g_I^2(v_1v_3+n_1n_2+n_1'n_2')+\frac{25}{18}g_{Y'}^2n_1n_2,\\
S_{46} &= \lambda\lambda'(v_2^2+v_3^2)+\frac{1}{2}g_I^2(n_1n_1'+n_2n_2')+\frac{25}{18}g_{Y'}^2n_2n_2'-m_3^2,\\
S_{47} &= -\lambda\lambda' v_1v_3+\frac{1}{2}g_I^2(n_1n_2'-n_2n_1')+\frac{25}{18}g_{Y'}^2n_2n_1',\\
S_{55} &= \lambda^2(v_1^2+v_2^2)+\frac{1}{2}(g_I^2+\frac{25}{9}g_{Y'}^2)n_1^2+\frac{1}{4}g_I^2(v_3^2-v_1^2+n_1^2+n_2^2+{n_1'}^2-{n_2'}^2)\\
&\quad -\frac{5}{36}g_{Y'}^2V^2+m_N^2,\\
S_{56} &= -\lambda^2v_1v_3+\frac{1}{2}g_I^2(n_2n_1'-n_1n_2')+\frac{25}{18}g_{Y'}^2n_1n_2',\\
S_{57} &= \lambda\lambda'(v_1^2+v_2^2)+\frac{1}{2}g_I^2(n_1n_1'+n_2n_2')+\frac{25}{18}g_{Y'}^2n_1n_1'-m_3^2,\\
S_{66} &= \lambda'^2(v_2^2+v_3^2)+\frac{1}{2}(g_I^2+\frac{25}{9}g_{Y'}^2){n_2'}^2+\frac{1}{4}g_I^2(v_1^2-v_3^2+{n_1'}^2+{n_2'}^2-n_1^2+n_2^2)\\
&\quad -\frac{5}{36}g_{Y'}^2V^2+m_{N'}^2,
\end{aligned}
$$

$$
\begin{aligned}
S_{67} &= -\lambda'^2 v_1 v_3 + \frac{1}{2} g_I^2 (v_1 v_3 + n_1 n_2 + n_1' n_2') + \frac{25}{18} g_{Y'}^2 n_1' n_2', \\
S_{77} &= \lambda'^2 (v_1^2 + v_2^2) + \frac{1}{2}(g_I^2 + \frac{25}{9} g_{Y'}^2) {n_1'}^2 + \frac{1}{4} g_I^2 (v_3^2 - v_1^2 + {n_1'}^2 + {n_2'}^2 + n_1^2 - n_2^2) \\
&\quad - \frac{5}{36} g_{Y'}^2 V^2 + m_{N'}^2.
\end{aligned} \tag{28}
$$

B.2 Pseudocalar Higgs boson mass-squared matrix

The mass-squared matrix for pseudoscalar Higgs bosons is also a 7×7 symmetric matrix, P.

$$
\begin{aligned}
P_{11} &= (\lambda n_1 + \lambda' n_1')^2 + (\lambda^2 + \lambda'^2) v_2^2 + \frac{1}{4} g_L^2 (v_1^2 - v_2^2 + v_3^2) + \frac{1}{4} g_Y^2 (v_1^2 - v_2^2 + v_3^2) \\
&\quad + \frac{1}{4} g_I^2 (v_1^2 + v_3^2 + n_2^2 - n_1^2 + {n_2'}^2 - {n_1'}^2) + \frac{1}{36} g_{Y'}^2 V^2 + m_{\mathcal{H}}^2, \\
P_{12} &= \lambda A n_1 + \lambda' A' n_1', \\
P_{13} &= -(\lambda n_1 + \lambda' n_1')(\lambda n_2 + \lambda' n_2') + \frac{1}{2} g_I^2 (n_1 n_2 + n_1' n_2'), \\
P_{14} &= \lambda v_3 (\lambda n_1 + \lambda' n_1') - \frac{1}{2} g_I^2 v_3 n_1, \\
P_{15} &= -\lambda v_3 (\lambda n_2 + \lambda' n_2') + \frac{1}{2} g_I^2 v_3 n_2 + \lambda A v_2, \\
P_{16} &= \lambda' v_3 (\lambda n_1 + \lambda' n_1') - \frac{1}{2} g_I^2 v_3 n_1', \\
P_{17} &= -\lambda' v_3 (\lambda n_2 + \lambda' n_2') + \frac{1}{2} g_I^2 v_3 n_2' + \lambda' A' v_2, \\
P_{22} &= (\lambda n_1 + \lambda' n_1')^2 + (\lambda n_2 + \lambda' n_2')^2 + (\lambda^2 + \lambda'^2)(v_1^2 + v_3^2) - \frac{1}{4} g_L^2 (v_1^2 - v_2^2 + v_3^2) \\
&\quad + \frac{1}{4} g_Y^2 (v_1^2 - v_2^2 + v_3^2) + \frac{1}{9} g_{Y'}^2 V^2 + m_{H_2}^2, \\
P_{23} &= -\lambda A n_2 - \lambda' A' n_2', \\
P_{24} &= \lambda A v_3, \\
P_{25} &= -\lambda A v_1, \\
P_{26} &= \lambda' A' v_3, \\
P_{27} &= -\lambda' A' v_1, \\
P_{33} &= (\lambda n_2 + \lambda' n_2')^2 + (\lambda^2 + \lambda'^2) v_2^2 + \frac{1}{4} g_L^2 (v_1^2 - v_2^2 + v_3^2) + \frac{1}{4} g_Y^2 (v_1^2 - v_2^2 + v_3^2)
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{1}{4}g_I^2(v_1^2+v_3^2+n_1^2-n_2^2+{n'_1}^2-{n'_2}^2)+\frac{1}{36}g_{Y'}^2V^2+m_{\mathcal{H}}^2, \\
P_{34} &= -(\lambda n_1+\lambda' n'_1)\lambda v_1+\frac{1}{2}g_I^2 v_1 n_1-\lambda A v_2, \\
P_{35} &= (\lambda n_2+\lambda' n'_2)\lambda v_1-\frac{1}{2}g_I^2 v_1 n_2, \\
P_{36} &= -(\lambda n_1+\lambda' n'_1)\lambda' v_1+\frac{1}{2}g_I^2 v_1 n'_1-\lambda' A' v_2, \\
P_{37} &= (\lambda n_2+\lambda' n'_2)\lambda' v_1-\frac{1}{2}g_I^2 v_1 n'_2, \\
P_{44} &= \lambda^2(v_2^2+v_3^2)+\frac{1}{4}g_I^2(v_1^2-v_3^2+n_1^2+n_2^2-{n'_1}^2+{n'_2}^2)-\frac{5}{36}g_{Y'}^2V^2+m_N^2, \\
P_{45} &= -\lambda^2 v_1 v_3+\frac{1}{2}g_I^2(v_1v_3+n'_1n'_2), \\
P_{46} &= \lambda\lambda'(v_2^2+v_3^2)+\frac{1}{2}g_I^2 n_1 n'_1-m_3^2, \\
P_{47} &= -\lambda\lambda' v_1 v_3-\frac{1}{2}g_I^2 n_1 n'_2, \\
P_{55} &= \lambda^2(v_1^2+v_2^2)+\frac{1}{4}g_I^2(v_3^2-v_1^2+n_1^2+n_2^2+{n'_1}^2-{n'_2}^2)-\frac{5}{36}g_{Y'}^2V^2+m_N^2, \\
P_{56} &= -\lambda^2 v_1 v_3-\frac{1}{2}g_I^2 n_2 n'_1, \\
P_{57} &= \lambda\lambda'(v_1^2+v_2^2)+\frac{1}{2}g_I^2 n_2 n'_2-m_3^2, \\
P_{66} &= \lambda'^2(v_2^2+v_3^2)+\frac{1}{4}g_I^2(v_1^2-v_3^2+{n'_1}^2+{n'_2}^2-n_1^2+n_2^2)-\frac{5}{36}g_{Y'}^2V^2+m_{N'}^2, \\
P_{67} &= -{\lambda'}^2 v_1 v_3+\frac{1}{2}g_I^2(v_1v_3+n_1n_2), \\
P_{77} &= \lambda'^2(v_1^2+v_2^2)+\frac{1}{4}g_I^2(v_3^2-v_1^2+{n'_1}^2+{n'_2}^2+n_1^2-n_2^2)-\frac{5}{36}g_{Y'}^2V^2+m_{N'}^2. \qquad (29)
\end{aligned}
$$

B.3 Charged Higgs boson mass-squared matrix

The mass-squared matrix for charged Higgs bosons is a 3×3 symmetric matrix, C.

$$
\begin{aligned}
C_{11} &= (\lambda n_1+\lambda' n'_1)^2+\frac{1}{4}g_L^2(v_1^2-v_2^2+v_3^2)+\frac{1}{4}g_I^2(v_1^2+v_3^2+n_2^2-n_1^2+{n'_2}^2-{n'_1}^2) \\
&\quad +\frac{1}{4}g_Y^2(v_1^2-v_2^2+v_3^2)+\frac{1}{36}g_{Y'}^2V^2+m_{\mathcal{H}}^2, \\
C_{12} &= (\lambda^2+{\lambda'}^2)v_1v_2-\frac{1}{2}g_L^2 v_1 v_2+\lambda A n_1+\lambda' A' n'_1, \\
C_{13} &= -(\lambda n_1+\lambda' n'_1)(\lambda n_2+\lambda' n'_2)+\frac{1}{2}g_L^2 v_1 v_3+\frac{1}{2}g_I^2(v_1v_3n_1n_2+n'_1n'_2),
\end{aligned}
$$

$$
\begin{aligned}
C_{22} &= (\lambda n_1 + \lambda' n_1')^2 + (\lambda n_2 + \lambda' n_2')^2 + \frac{1}{4} g_L^2 (v_1^2 + v_2^2 + v_3^2), \\
&\quad - \frac{1}{4} g_Y^2 (v_1^2 - v_2^2 + v_3^2) + \frac{1}{9} g_{Y'}^2 V^2 + m_{H_2}^2, \\
C_{23} &= (\lambda^2 + {\lambda'}^2) v_2 v_3 - \frac{1}{2} g_L^2 v_2 v_3 - \lambda A n_2 - \lambda' A' n_2', \\
C_{33} &= (\lambda n_2 + \lambda' n_2')^2 + \frac{1}{4} g_L^2 (-v_1^2 + v_2^2 + v_3^2) + \frac{1}{4} g_I^2 (v_1^2 + v_3^2 + n_1^2 - n_2^2 + {n_1'}^2 - {n_2'}^2) \\
&\quad + \frac{1}{4} g_Y^2 (v_1^2 - v_2^2 + v_3^2) + \frac{1}{36} g_{Y'}^2 V^2 + m_{\mathcal{H}}^2. \qquad (30)
\end{aligned}
$$

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